“Dynamic Position Control of Underwater Robot using Model Predictive Controller”

Abstract

This work concentrates on dynamic positioning control for underwater robot both considering its advantages and disadvantage, motivated by the fact that there are uncertainties and external disturbances, and the situations which underwater robot often copied with nonlinear dynamics. Underwater robotics systems are becoming critical in the world for different purposes including exploration, environmental surveys, and offshore activities. But the marine environment is unfavorable since controlling a ROV introduces hydrodynamic forces, fluctuations in buoyancy and external interference such as currents. In fact, basic strategies of control including PID control and sliding mode control have limitations in providing stability and accuracy in work that needs to deal with constantly changing conditions. Such methods are incapable of handling non-linearity and are often slow to update in a real-world operation causing positional drifts and operational inefficiencies. To overcome these limitations, this research utilizes Model Predictive Control (MPC), a complex and highly efficient control algorithm that can predict and manage the system dynamics on a finite control horizon. In more detail, an LMPC framework is designed to guarantee stability while accurately positioning a robot in compliance with specified control inputs. To satisfy the kinematics and kinetics requirements, the study employs Fossen's dynamic model that exactly describes rigid body dynamics, hydrodynamic influence and external forces on the robot. A Lyapunov function is used to achieve stability of the system and securely keep the robot at the desired position despite interferences. Besides augmenting the original problem of trajectory tracking, the LMPC framework also addresses the issue of lack of robustness arising from uncertainties in the environment. The methodology includes the establishment of an underwater robot control system and its subsequent modelling through simulation on MATLAB. The LMPC framework is tested under two scenarios: quiet and non-noise conditions and the presence of noise factors. In one of the simulations without noise, the controller exhibits tight control in tracking a specific path, stability and low positional error. The proposed LMPC remains efficient in noisy environment and disturbances; in several control cycles it brings the robot back to the track of the desired movement. As such, these results demonstrate the versatility of LMPC against real-life underwater enshrinements.

Thus this research is of great significance to the field of underwater robotics due to the proven efficiency of LMPC over conventional methods of control. MPC capabilities for change anticipation involve its ability to adjust control actions before they are triggered due to their predictive nature, and Lyapunov stability in providing consistent control performance under dynamic conditions. These results highlight the capability of LMPC in enhancing the performance, precision, and reliability of underwater robots in uses ranging from exploration of the deep sea to environmental monitoring and offshore maintenance. Despite the results, this study has its own drawbacks. The tests are performed assuming that reality is perfect and various real conditions might increase some factors that the model does not take into account. The following constituted the limitations of the work Hence, future work will aim at solving these limitations through improving the computational optimality of MPC for real-time computation. In addition, applying machine learning techniques to the LMPC framework may also help in the improvement of adaptability and decision making in scenarios where LMPC is to be implemented. Field test is also required to establish that the proposed control strategy is realistic and can be used in the actual environment.

Finally, this study proves LMPC as a feasible and efficient control approach in underwater robots and creates a foundation on which further improvements of autonomous marine vehicles can have built upon. Overcoming the issues of the nonlinear dynamics and disturbances in the environment of application the present study opens a path to enhance the reliability and effectiveness of the underwater operations and thus to the development of this area of the underwater robotics.

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CHAPTER NO. 01: INTRODUCTION

1.1 Introduction

Underwater robotics has gain significant importance over the past several decades due to increasing demand for deep sea exploration. The underwater robotics started back in 1950s when first remotely operated vehicle were developed for the scientific and military purpose. These early remotely operated vehicle were designed especially for surveillance and recovery of object from the ocean surface.

One of significant milestone came in late 1950s with the development of an underwater robot which were used by French navy for mine detection. This was later followed by US navy and launched a remotely wire controlled robot in 1960s which were used for cable repairs. This repairing robot gain a lot of attention later in 1966 when it was used to recover a hydrogen bomb lost after a midair collision. This success highlighted the potential of underwater robot for performing very critical operations in the ocean [5, 14].

In 1970s a very rapid advancement occurs in underwater robotics technologies for ocean research and offshore oil exploration. Later underwater robot become a standard tool for the exploration of the ocean and other operations. Initially the robot was controlled re- motely but after some time concept of autonomous robot came into view which is capable of operating without need of any operator. The research began on the autonomous robot to operate them autonomously which majorly rely on preprogrammed instruction and integrated sensors to perform the complex missions.

The first autonomous robot was developing in early 1980s by the French researcher named Epaulard for marine exploration. It was designed for scientific exploration and it has au- tumorous navigation capabilities. Later a lot of research were done in development of underwater robotics with integration of sensors for better performance [3]. As remotely operated and autonomous robot became more sophisticated, their usage became diverse in multiple applications like environmental monitoring, marine exploration etc. One of most famous mission done by scientist and researcher was exploration of titanic wreck. The robot used were equipped with high quality camera and robotics arms. This further make the underwater robotics an area of interest for researcher and scientist.

By the early 2000s, advancement in sensor technology made the capabilities of underwater robot more efficient and reliable. After the more enhancement in underwater robots, it was widely used for environmental monitoring and military applications ranging from monitoring ocean and to underwater detection.

Today robot is used in wide range of operations in scientific and other purposes. As the demand for deep sea exploration increases, underwater emerged as a key source for this and play a critical role in performing these complex challenges.

1.2 Background Knowledge

The ocean exploration is a great interest for military and scientific purpose. Extreme pres- sure, low visibility, high corrosion and other limitations make the ocean a difficult area to explore. With time these challenges were overcome when the first ships sailed. But nowadays, underwater robotics has become an area of challenge for the scientist [5, 6].

Initially, remotely operated robots have been used for underwater exploration. A skilled person was required to operate these robots and dedicated vessel were required too for power and communication between operator and robot. But now Autonomous underwater robot is used for the marine exploration. This come with wireless communication and preprogrammed feature.

The autonomous nature of robots need a high level of robustness, navigation, guidance and control. These challenges arise due to the unpredictable nature of the ocean. There are many challenges face by robots and a critical component is the ability of robot to maintain its position and orientation during submarine operations which is also known as dynamic positioning.

The dynamic positioning control of underwater robots has become an area of interest for the researchers due to its critical role in multiple applications like underwater environmental monitoring, marine exploration and data collection. As we all know that ocean is covering over 70% of the earth surface area which also holds many resources as well as ecological area which remained unexplored. To access these resources and gather data from unexplored area of the ocean we must operate the underwater vehicles and robots very effectively and with high efficiently [14].

Recent studies regarding the implementing MPC in the underwater robots has indicates that MPC can lead the robot to better, efficient and reliable operations against the dynamic changes occurring continuously in the underwater [11,14,19]. A study in 2021 demonstrated that MPC could work effectively and reduce the positional drift in the complex scenarios. So overall MPC enables underwater robots to execute the task with more precision and making it more beneficial in the field of the underwater robotics.

The need for this research is highlighted by demonstrating the increasing efficiency and effectiveness of MPC in underwater applications. As researchers are getting deeper into the investigation of complexities and exploration of underwater environments using MPC as control strategy presents a reliable solution that not only overcome the challenges of underwater environment but also provide a way for future enhancement. The capability to respond to changing dynamic conditions is crucial for improving the safety, reliability and efficiency of the underwater robots which ultimately results in more successful exploration and data collection missions.

To summarize the motivation behind this research is critical to enhance the underwater robot through enhanced dynamic positioning control. By focusing on developing and analysing an MPC framework, this study aims to provide significant insights into control methodologies that will enhance the capabilities of underwater robots. So these advancements will not only deepen our understanding of the robotics and control systems but also provide more efficient solution for the utilization of marine resources.

1.3 Motivation

A significant amount of effort has been made to study the underwater robotics and their control. Due to its complexity, there are still control problems in it which needed to be solved. These underwater robots face unpredictable environment like continuously changing drag and buoyancy forces.

Conventional control like proportional, integral and differential falls behind in the continuously changing environment, resulting in positional drift and inefficiencies. This deficiency not only affect the precision but also effect the safety and operation of robot system. So with the increasing demand for underwater exploration and operations, there is also need of a better sophisticated control strategy that perform the better operation to the dynamic environment.

The need for this research is highlighted by demonstrating the increasing efficiency and effectiveness of MPC in underwater applications. As researchers are getting deeper into the investigation of complexities and exploration of underwater environments using MPC as control strategy presents a reliable solution that not only overcome the challenges of underwater environment but also provide a way for future enhancement. The capability to respond to changing dynamic conditions is crucial for improving the safety, reliability and efficiency of the underwater robots which ultimately results in more successful exploration and data collection missions.

As MPC is a powerful tool for solving nonlinear complex systems so this work focuses on developing a Model Predictive Control system for dynamic positioning of an underwater robot. First an MPC algorithm will be developed to help the robot to maintain its position in aquatic environment. Then further external forces challenges will be solved.

The motivation of this thesis mainly lies in the potential to enhance the performance and efficiency of the underwater robot. By developing an effective and robust control technique, we can improve the accuracy and reliability of the underwater robot in various operations.

To summarize the motivation behind this research is critical to enhance the underwater robot through enhanced dynamic positioning control. By focusing on developing and analysing an MPC framework, this study aims to provide significant insights into control methodologies that will enhance the capabilities of underwater robots. So this advancement will not only deepen our understanding of the robotics and control systems but also provide more efficient solution for the utilization of marine resources.

1.4 Significance

This project is significant because the precise positioning of underwater the robots is essential for various applications, including underwater exploration, marine research and underwater data collections. With the rising demand of underwater operations, the ability to maintain accurate positioning become increasingly necessary to play vital role for the both safety and effectiveness of robot. Unfortunately, existing control methods often falls short in dynamics environments, where unpredictable challenges and disturbances can cause significant positional drift and reduce robot operational efficiency.

- **Addressing Technologies Gap:** Traditional dynamic control method faces challenges such as unpredictable nature and fluid dynamics complexities. By proposing an advance control framework based on model predictive control, this research will address many limitations and leading to more robust and adaptable control for precise positioning.

- **Improving Control System:** Accurate dynamic positioning is required for performing critical tasks for scientific and military purposes. This innovative approach is expected to improve the control precision which enables the underwater robot to perform complex challenges and maintain their position effectively even in challenging condition.

- **Enhancing Efficiency:** By integrating a better control technique, the system will minimize energy consumption and reduce operational cost will make it more efficient and reliable. Efficiency is key factor for long period of operations where battery life and resources management is very critical.

- **Expanding Application Scope:** The advancement in dynamic positioning control can broaden the scope of underwater robots in the field of hazardous environment such as rescue operations and marine explorations where position is a key factor.

- **Academic Contribution:** The advancement in this topic will contribute to the academic and engineering knowledge base in control system and underwater robotics. By developing and implementing a better control strategy, this study will provide valuable insight in future research and development in the field.

- **Enhancing Autonomous Operation:** The development of advance control for dynamic position will enhance the functionality and operation of the underwater robot, allowing them to perform independently without operator need. This will help in long duration mission where robot need to operate reliably over extended period.

- **Human Risk Reduction:** By improving the position control and reliability of underwater robot, this will become source for the complex and dynamic operations that is difficult to perform by human. This study will also contribute to enhancing safety of operation.

Furthermore, this research is not limited to just making precise positioning more efficient or effective instead the improved positioning accuracy can result in more effective data collection in the marine biology, improved monitoring of underwater environment and enhance the safety of the robot. This is due to the MPC control methodology in underwater robotics. And make the application broader.

1.5 Objectives

The primary objective of this research is to develop and implement a MPC framework for improving the dynamic position control of underwater robots. This study aims to focuses on the challenges faced for maintaining precise position in the presence of continuously changing unpredictable currents, velocities and disturbances that effects the stability and accuracy of robot in presence of the conventional control methods. By utilizing MPC, this research aims to optimise the control strategy to ensure faster and better response in maintaining the robot position with minimum drift [1,2,3]. Through the design and implementation of an MPC controller, the research will demonstrate an improved control performance compared to traditional control like PID etc.

This work focuses on developing a Model Predictive Control system for dynamic positioning of an underwater robot. first an MPC algorithm will be developed to help the robot to maintain its position in aquatic environment.

Ultimately this study aims to uses the model predictive control for addressing the limitations of existing conventional control methods and providing an effective solution to overcome the challenges occurring continuously in the ocean. Conventional control strategies have no prediction capability while MPC offers real time adjustment based on predictions of the robot Behaviour and surrounding environmental factors. Allowing the system to mitigate the disturbances. This capability of MPC not only enhances the stability but also improve the overall efficiency of underwater robot, enabling them to perform tasks with more reliably and effectively.

1.6 Problem Statement

Underwater robot plays a vital role in various application like deep sea exploration, environment monitoring and marine exploration. The ability of robot to maintain stable and precise positioning in dynamic environment is essential for these operations. However, underwater environment is challenging due to unpredictable factors like hydrodynamic force and other dynamic changes. These nonlinear disturbances effects on the robot dynamics and make precise positioning more challenging.

So controlling the accurate position of an underwater robot in a dynamic environment become more crucial and challenging due to multiple factors such as water changing pressure, nonlinear forces and changing currents. These changes and disturbances cause fluctuation, affect the stability and accuracy of robot which plays and important role in multiple applications like underwater exploration, maintenance and monitoring [14].

Traditional control strategy faces difficulties in achieving optimal performance in high dynamic conditions, especially while managing multiple degree of freedom systems like underwater robots. Traditional control method such as PID and Sliding mode controller fall behind due to complex dynamics and uncertainties faced in underwater navigation. As a results they may cause poor performance in maintaining desired position especially under varying environmental conditions. These limitations need to be addressed by more advance control technique that provide better robustness and adaptability to disturbances [5].

Model Predictive Control offers a reliable solution to this problem by obtaining a mathematical model of underwater robot dynamics to predict future Behaviour and optimise control input in real time monitoring. MPC potentially enhance the position performance of under- water robots in dynamic environment.

This research aims designing a Model Predictive Control for a multi-dimensional underwater robot. The primary aim is to model robot dynamics, simulate its performance in MATLAB with implementation of MPC to achieve precise positioning control despite nonlinearities and external disturbances.

1.7 Scope and Limitation

This research will make a significant contribution to the field of underwater robotics by providing the potential of MPC as an advanced control strategy for the dynamic position of underwater robot in the complex dynamic environment. The key outcome of this re- search lies in addressing the unique challenges faced by the environment due to the dynamic environment and external disturbances such as hydrodynamic forces and buoyancy force.

This scope of study on dynamic position control of underwater robots comprises on several key factors. One of major key factor is technologies development to enhance the dynamic positioning capabilities of the robot. Another key factor is the potential for future research and implementation of hardware for the control system being developed in this study.

Further this research focuses on the designing, developing and implementing of model predictive control (MPC) controller for the underwater robotics systems especially for the precise positioning control under various aspects of underwater environmental conditions. Moreover, this project will cover mathematical modelling of the robot, designing of model predictive control and the validation of the performance and its results through Matlab.

The limitations of this study include: Software based analysis may be differing from real world scenarios and conditions due to ideal assumptions are made in the modelling and de- signing process. Due to uncertainties in environmental factors like rapidly changing occurring in the currents or the extreme weather conditions may not be fully accounted for.

This study does not cover the hardware implementation or any real world testing of the robotic system due to the both time constraints and resource limitations. The study does not encounter communication delays and data transmissions issues that happens during real world testing in unpredictable disturbance occurring in underwater environment. The limitations of this study is given as:

- Software based analysis may be differing from real world scenarios and conditions due to ideal assumptions are made in the modelling and designing process.
- Due to uncertainties in environmental factors like rapidly changing occurring in the
- currents or the extreme weather conditions may not be fully accounted for.
- This study does not cover the hardware implementation or any real world testing of the robotic system due to the both time constraints and resource limitations.
- The study does not encounter communication delays and data transmissions issues that happens during real world testing in unpredictable disturbance occurring in underwater environment.

1.8 Summary

In summary, this research offers a comprehensive and novel approach to overcome the challenges faced by the robot in maintaining precise position. By demonstrating MPC effectiveness, it will pave a more efficient, reliable and adaptive control in the domain of control systems. This will significantly increase the area of interest in the future of autonomous robots. The significance of this research is toward the advancement of robotics operation and challenges. This is essential for application in various industries like offshore exploration, underwater construction and marine exploration. Secondly improved dynamic can enhance the operations and enabling effective monitoring of the marine ecosystems.

One of major contribution of this work is the development of a robust MPC algorithm for underwater robots. This algorithm will be focusing on the challenges faced to improve positioning accuracy by compensating the dynamic disturbances. The research focuses on the adaptability control system, allowing the underwater robot to perform effectively and reliably in complex dynamic environment. Moreover, the dynamic position control presents a complex and critical engineering chal-lenge, given the nonlinear and unpredictable nature of underwater environment. This re- search overcome the limitation of traditional control methods through the development and implementation of a robust based control system.

CHAPTER NO. 02: LITERATURE REVIEW

2.1 Background

Underwater robot is a vehicle designed operate in marine environment. These robotics systems are equipped with sensor and tools that enables them to perform multiple operation ranging from exploration of sea to data collection. Underwater robot is specially design to operate in challenging environment which is equipped with advance control techniques to navigate and interact with their environment. As technology continues to evolve, underwater robots are becoming very popular integrated with the advance control algorithm and sensors technology. By improving the control ability to navigate and manipulate in the surroundings, it will allow us to explore the things in better way in such a dynamic environment [33].

Underwater robots have become a very useful tool for a wide range of marine operations. These includes ocean surveys that help researcher to monitor the ocean surface, offshore oil and gas exploration, underwater construction such as building cable network, lay downing pipelines etc. The importance of all these operations in industries like marine research, de- fence, energy and environmental protection need a precise, reliable and robust control to ensure that underwater robot can operate effectively in the underwater challenging environment [9].

The control of underwater robot presents a significantly complex challenge compared to ground and aerial vehicles. Unlike their air and land based counterparts, underwater robot goes through a dynamic environment that is characterized by several unpredictable factors [11]. Underwater robot governed by nonlinear hydrodynamics where several forces acting on the robot vary continuously. Hydrodynamics forces like drag are difficult to predict because they depend on factors like robot shape, velocity and the surrounding water. Additionally, as robot move change in buoyancy force occur along with various other external disturbances which further complicate the control process. These challenges made very difficult to achieve the same level of precision and stability that is possible in land or aerial base systems. As we are in a new era of underwater exploration, the significance of control system for underwater robots cannot be ignored. It is challenging but it is a gateway to understand the ocean and marine ecosystem. The ongoing research in this field not only seek to redefine control strategy but focusing on exploration of aquatic environment.

2.2 Challenges and Control Approaches

Underwater robotics has gain a remarkable progress over the past few years. These advancements have enable the robots to perform complex tasks and operation with great precision. A critical aspect in it is to control and overcome the challenges I.e. dynamic changes and exter- nal disturbances. These environmental factors are changing rapidly and are unpredictable, making precise control a challenge for the researchers.

A major challenge faced in dynamic positioning of underwater robots is highly nonlinear and time varying nature of the system dynamics. The Behaviour of the robot is observing by complex hydrodynamics equations that give the relationship between the interactions of robot with surrounding water [13]. These equations are nonlinear because of the factors like buoyancy, drag force and other external disturbances which changes over the time. For example, a sudden change in the ocean current and forces speed cause a high deviation from the desire position making it a very hard challenge for the researchers. The ability to handles all these nonlinearities is such a critical environment is very challenging but also crucial for maintaining accurate and stable position control.

Traditional control methods like Proportional Integral Derivative (PID) control and sliding mode control have been widely used in underwater robot control system, PID control is one of most commonly technique which is use due to its simplicity and ease of implementation [24]. Continuously error is fed to the proportional, integral and derivative term to apply correction to maintain the desire position of robot. While PID is used where simple dynamics are there as its performance in case of nonlinear dynamics falls behind.

After traditional control, adaptive control [29] were came in use for underwater robot which control the parameters in real time based on observed system and Behaviour of the surrounding environment allowing the control to make an accurate decision for the control. However adaptive control also struggles to control to maintain the high efficiency and performance when dealing with the rapid and unpredictable underwater environment. Sliding mode controller which is known for robustness to disturbances can handle nonlinearities more effectively than PID control but it faces chattering due to rapid changes/oscillations which lead the robot to inefficient performance [16].

In response to them more advance control technique has been explored. Among them Model predictive control (MPC) has emerged as a valuable solution for the dynamic positioning of underwater robot [5,14]. MPC is an advance con troll strategy that overcomes many limitations of the traditional controllers. It operates by predicting the future Behaviour of the system over the time horizon and optimise the output accordingly. At every control step, MPC provides an optimise output which minimize the predefined cost function, which is typically for tracking errors and control effects. The optimization is performed in iterations so the control system handles the nonlinear dynamics.

For underwater robot, MPC predictive capability is valuable. Bu predicting the system future states, MPC can anticipate the effect of disturbances such as current or the external forces. This ability allows MPC to maintain more accurate and stable movement control than the conventional method like PID and sliding mode controller. Furthermore, MPC is well suited for more degree of freedom unlike PID require to be designed separately for each degree of freedom.

In summary, dynamic position control of underwater robots is complex and challenging problem due to highly nonlinear nature of the underwater environment. Traditional control methods are also useful but they fall short in handling full range of disturbances and nonlinearities. MPC provide a valuable solution to all dynamic changes as it has potential for real time application further make it promising solution as a leading control strategy in the field of underwater robotics.

2.3 Model Predictive Control Overview

Model predictive control is a cutting edge control strategy that has gain significant importance for its ability to handle complex, nonlinear system especially in dynamic environment where continuous changing is occurring.

Unlike traditional controllers, such as PID which is based on reactive approach to error, MPC uses a future prediction methodology. It uses the mathematical model of the system to predict its future Behaviour over a finite time horizon. At each time instant, MPC do optimization that minimize the cost function and it provide a balance output between energy consumption and performance.

A key feature of MPC is its ability to handle multi input multi output systems, where multiple inputs are used to control multiple outputs. Many robotics falls in this category of multi input multi output, due to their complex dynamics and the need to perform sudden required operations simultaneously. Additionally, MPC consider the input explicitly to ensure that systems operate within safe and efficient range within the dynamic condition.

The predictive nature of MPC means that it is capable of recognizing and compensating the disturbances before they impact the system. In dynamic environments like aquatic environment underwater robot predictive capability become very crucial. Hydrodynamic drag, buoyancy force and changing current require real time adjustment for maintaining stable control. By continuously recalculating control actions based on the dynamic environment and making future prediction states allows the system more stable even in highly uncertain conditions.

MPC heavily relies on the accuracy of the system model which is used for future prediction and any mistake will lead to instability in the control decision. In underwater environment where hydrodynamic forces and environmental disturbances are difficult to model precisely so this can be a significant obstacle. Despite this MPC control have gain a rapid growth in the control system, with its numerous benefits and it’s potential to improve control performance in every aspect.

Researcher are continuously exploring the way to enhance the real time feasibility of MPC through the adaptive control and distributed control technology. As the modelling technique for MPC will improves, it will become the control method choice for many applications.

2.4 MPC Applications in Underwater Robots

Dynamic positioning plays crucial role in the underwater robotics. Precise position is the ability to maintain a stable position and orientation while responding to environmental disturbances like drag and buoyancy force. This ability of robot plays vital role in marine application including offshore oil extraction, underwater construction and environmental monitoring. Previous studies have proven the effectiveness of MPC in dynamic positioning task for underwater robots [44].

In 2008 a studied was carried out for underwater robot trajectory tracking using MPC which highlights the significance of MPC over the traditional control methods [24]. But later on PID, LQR and sliding mode controller are still in use. After that, in 2019 LQR and H infinity control method have been employed to achieve better robustness and stability in underwater robot dynamics. However, these methods were performing better comparatively to the PID but again there were few limitations and challenges in the real world scenarios [16].

Another approach i.e. sliding mode controller were used in 2020 which provide robustness again uncertainties and external disturbances. While sliding mode controller offer advantage in term of robustness but chattering effect were induced in this due to continuous external changes and it cause negative impact on the performance of underwater robots [23].

In the recent years, Model Predictive Control has gained a significant importance over the other control strategies. In 2021 and 2022 many researchers conducted research on it and concluded that it is significantly improving the mission success rate. The application of MPC is not only challenging but are too complex for the real time optimization.

Furthermore, the research was conducted to integrate the sensors for the real time monitoring and data collection from the underwater aquatic environment. These finding also concluded that the model predictive control is working efficiently in the continuously changing environment and also improving the mission success rate.

Overall, the traditional control methods like PID and LQR have been contributed significantly in to the underwater robot systems, but they offer very limitation in the continuous changing nature. Advance control techniques such as Model predictive control offer an alternative control to the PID and LQR with the ability to predict future situations, higher response time and their overall robustness. By integrating environmental data in to MPC, a highly improved performance is obtained [5].

2.5 Summary

The exploration of underwater robotics controlled through Model predictive control has gained significant attention due to unique challenges and opportunities presented by under- water robotics. This literature review provides a comprehensive approach for various aspect related to the control strategies, modelling approaches and application of MPC in underwater robotics.

The review provides various modelling approaches, emphasizing on the significance of accurate representation of underwater robot dynamics modelling. All famous and widely used model were briefly discussed and a model which highlighted the applicability in multiple models.

This review also compared multiple control strategies and giving a promising solution to the challenges faced by underwater robotics. MPC ability to predict future state and adjust control action accordingly and makes model more suited for the positioning control of underwater robotics.

In summary MPC has emerged as a better and efficient control technique in the field of underwater robotics. MPC is more capable of managing complex, multi variable task while optimising control input in real time. Despite the challenges face during the computation and implementation, ongoing has introduced more efficient algorithm and integration of machine learning offer more exciting technique to get more enhanced performance.

As underwater exploration and operation have continuously gain more popularity in the recent year. By addressing the identified research gap and obtaining precise control, MPC will be used to play vital role in advancing in the field of robotics. This integration will help the robotics field to gain more capacity for tackling the most marine environment challenges.

CHAPTER NO. 03: SYSTEM MODELLING

3.1 Background

In the period of World War II, state space approach was introduced and gain a significant development. This technique involves modelling a system and developing a mathematical model to represent that system through first order differential equations. Then the controller is designed to meet the specific characteristics of the system. within the framework of this thesis, the dynamic model consists of set of first order differential equations which describe the motion of vehicle [1].

Submarines standard equation of motion were published in 1967 by Gertler. These equations were revised by Feldman in 1979. These equation of motion are highly accurate due to extent to thorough treatment of hydrodynamic coefficients. This lead to the advancement in hydrodynamics modelling. Unlike submarines, underwater robots operate autonomously so robust and accurate control is mandatory. Some of most important assumptions or considerations for the robot is, high speed is not required, control is main purpose and applicability of modern control method is important and no experimental test are possible in this timeline.

It is clear that these standard equations for motion are not the answer to our modelling of robots. In modelling and simulation of the underwater robot, Humphrey's, Nahon's and Fossen's models are very popular [14]. Each model has its own focus and approach. Here is a brief overview of each model in system designing:

1. **Humphrey's Model**

    - **Focus:** The Humphrey model focus on dynamic of underwater vehicle in terms of their kinematic and dynamic Behaviour. It is often used for modelling remotely controlled vehicle and its provide relationship between the vehicle motion and force acting on it in the dynamic environment [14].

    - **Application:** This model is useful where precise position and orientation require so this make the robot to operate smoothly for dynamic positioning and navigation tasks.

2. **Nahon's Model**

- **Focus:** Nahon model revolves around the developing of mathematical model that provides complex interaction between the vehicle and the surrounding fluid medium. His model includes hydrodynamic forces and moments which provide insight into vehicles stability [14].
- **Application:** Nahon model is useful for analysing and simulating underwater vehicle to control input especially in planning and trajectory optimization tasks and dynamic positioning control [14].

3. **Fossen's Model**

- **Focus:** Fossen model is widely used model in the field of underwater vehicle as it offers comprehensive modelling technique for underwater vehicle. His model also consists of kinematic and dynamics modelling which provide a detailed framework for the modelling of motion of underwater vehicle in all motions along three dimensional axes [14].
- **Application:** Fossen mode is particularly useful for designing control systems so it is widely in use while implementing PID, Sliding mode controller and MPC techniques. More application includes dynamic positioning, path planning and control system designing.

All models contribute a lot in the modelling of underwater vehicle. Each model provides a unique insight into the dynamics and control of underwater robot. Fossen model is widely use because it contains both kinematic and dynamic modelling along with that its offer a robust framework for modelling robot across all degree of freedom. Here Fossen model will be used for designing of advance control system which will further enhance the robot ability to perform better and efficiently in the dynamic environment [15]. It is necessary to take a reference frame to describe the motion of underwater robot accurately. These reference frame serves as a coordinate system from which robot position and motion is measured. Here's the reference frame in our study:

Local North East Down reference frame (NED): This frame is useful for describing the position and motion of the robot, helping in position and motion control [5].

Body fixed reference frame: This frame is helpful for control purposes, allowing us easier computation of force and velocity which is directly related to robot current state [5].

Using these reference frame in Fossen model enhance the modelling and control of underwater vehicle by providing a local and global perspective on motion. This dual frame approach is essential for modelling the tasks for dynamic positioning and navigation and ultimately leading to more effective control strategy for the underwater robotics.

3.2 Robot Motion

Underwater robot operates remotely or autonomously but understanding the robot motion is very crucial for designing the control system to achieve the better and reliable performance. The motion of an Underwater robot is a fundamental concept in the robotics theory because it's define the robot capability to move in the environment but robot motion in the ocean is a complex topic to understand because it involve the mechanics of robot as well as its interaction with the fluid which significantly affects the motion of robots [1].

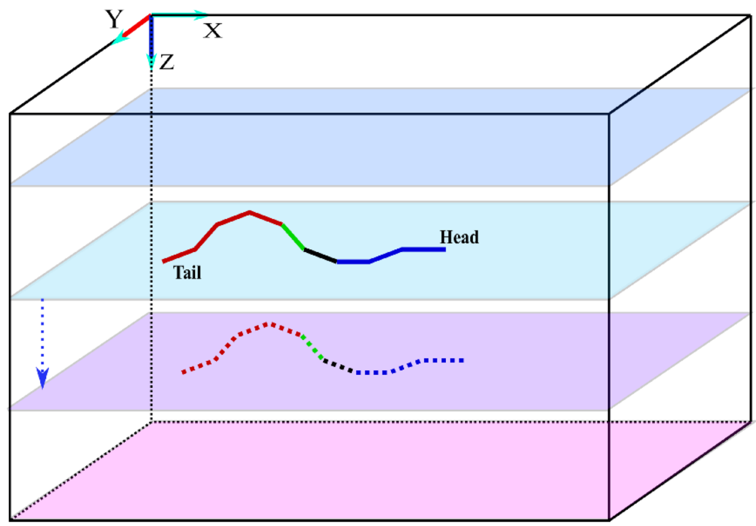

Figure 1: 3D motion schematic of a robot in underwater conditions[47]

Typically, robot motion is across three dimensional axes which is further categories in two categories: translational and rotational motion. Understanding these motions is necessary for precise navigation particularly in very complex nature of unpredictable environment. Here is the detailed explanation of key aspect of robot motion [33]:

3.2.1 Translational Motion

Translational motion describes the robot ability to move in three main axes i.e. *x*,*y* and *z*. These axes correspond to forward, backward, up, down, left and right motion. These motion is essential to navigate robot in complex environment for maintaining precise position and perform various tasks.

- **Surge** is the forward backward motion along the *x* axis which provide movement from the front to the back. Surge is a primary mode for moving robot in straight line. This motion is required to cover long distance and to reach specific target. In dynamic positioning, it is an important to do fine adjustment for holding position at a given location.

- **Sway** is the left-right motion along the *y* axis which provide movement from one side to other. Unlike surge which is directional, sway allow the robot to move laterally without changing its forward orientation. This movement is essential to avoid obstacles in such a dynamic environment. Sway is crucial for adjusting position of the robot without turning.

- **Heave** provide motion along the *z* axis in upward and downward direction. Heave is crucial for maintaining the depth of the robot and its control is necessary for maintaining a specific depth in applications like equipment installation or marine exploration [14].

Table 1: Translational Parameters for 4 DOF Underwater Robot

Terminology	**Motion**	**Position**	**Velocity**
Surge	x	P_x	u
Sway	y	P_y	v
Heave	z	P_z	w

3.2.2 Rotational movement:

Rotational movement is different from translation which allows the robot to rotate along the three axes to change its orientation.

- **Yaw** provides rotation around the *z* axis which allow the robot to rotate left and right. Yaw is important for maintaining the alignment of robot along designated path. Yaw play a crucial role in orientation especially when external forces like drag and buoyancy forces acts on it. By maintaining yaw along with longitudinal motion ensures a successful completion of mission which require accurate positioning.

Table 2: Rotational Parameters for 4 DOF Underwater Robot

Terminology	**Rotation**	**Angle**	**Velocity**
Yaw	z	P_ψ	r

Together these both translational and rotational motion is essential for performing complex tasks in maintaining stability in dynamic environment and achieving in precise positioning [4].

Robot position is describing by kinematic and dynamic model [1]. these model give relationship between the robot position, velocity, force and torque etc. These kinematic and dynamic model are crucial to calculate from the well-known model to get precise and better control. Further detailed discussion of these models is given below:

3.3 Kinematic Model

The accurate and effective modelling of an underwater robot is crucial for the dynamic position control systems. By representing the robot model, this provide a precise framework for analysing the performance and stability control algorithm for the better navigation in the marine environment. For a 4 degree of freedom underwater robot, the model consists of linear and angular movement i.e. surge, sway, heave and yaw. Here is the detailed focus on the following movements:

- Heave (Up and Down movement)
- Surge (forward and backward movement)
- Sway (left and right movement)
- Yaw (rotation along the vertical axis)

The vector representation of this will be:

$$\eta = \begin{bmatrix} x \\ y \\ z \\ \psi \end{bmatrix} \qquad (3.1)$$

This represent the position and orientation vector of the robot (position: *x*, *y*, *z*, and orientation: yaw *ψ*).

Now coming toward the velocity vector for 4 DOF underwater robot [5]. Each component of velocity corresponds to different type of motion. There are two main type of velocities. One is linear and the other is Angular. The Linear velocity provides movement to the robot along the x, y, and z axes while the angular velocity provides the rotation to the robot about

its axes. We can express the velocity vector as:

$$v = \begin{bmatrix} u \\ v \\ w \\ r \end{bmatrix} \qquad (3.2)$$

Where *u*, *v*, *w* is the surge, sway and heave velocity respectively and *r* provides the rotation about the vertical axis.

The rotational matrix represents rotatory motion along the axes in three dimensions. In 4 DOF, yaw is the only rotation that is occurring. As the object rotate along the z axis, its position is describing in cylindrical axis where z remains unchanged, x and y change the position according to the angle [5]. Hence the rotational matrix in the z axis is given as:

$$R_z(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.3}$$

Transformation matrix represents the relationship between the angular state and initial reference frame. The other term are omitted and by only considering yaw, the transformation

matrix is given as:

$$T_z = \begin{bmatrix} 1 & 0 & -\sin(\psi) \\ 0 & 1 & \cos(\psi) \\ 0 & 0 & 1 \end{bmatrix} \tag{3.4}$$

The Fossen's model of kinematic equation describe the robot kinematics and how the position and orientation change with respect to time. It can be expressed mathematically as:

$$\dot{\eta} = J(\eta)\nu \tag{3.5}$$

Where:

- $J(\eta)$ is the Jacobian transformation matrix, which maps the fixed body velocity to the inertial frame velocity.

$$J(\eta) = \begin{bmatrix} R_z(\psi) & 0_{3\times3} \\ 0_{3\times3} & T_z(\psi) \end{bmatrix} \tag{3.6}$$

3.4 Kinetic Model

Kinetics provides relationship between motion and force. An underwater robot is subject to some law of motion that consider the robot as rigid body. The robot is operating in the water so its experience hydro-static and hydrodynamics forces. So for robot mass, rigid body assumption is made and its eliminate the need to analyse between individual mass element of robot [1]. so the overall dynamics model of the system is given as:

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau + d_{ext} \tag{3.7}$$

Where M denotes 4x4 mass and inertia matrix. $C(v)$ denotes Coriolis and centripetal matrix. $D(v)$ denotes damping matrix which include hydrodynamic drag. d_{ext} represent the external disturbances. $\tau = [X, Y, Z, N]^T$ is the 4x1 control input vector which represents external force and moments acting on robot.

3.4.1 Rigid Body Dynamics

The rigid body assumption was made and the rigid body equation of motion can be ex- pressed in matrix form as:

$$M_{rbm}\dot{\nu} + C_{rbm}\dot{\nu} \tag{3.8}$$

where M_{rbm} is representing rigid body matrix of mass and C_{rbm} is representing Coriolis force matrix. For obtaining rigid body mass matrix lets define position vector for the center of gravity as:

$$\mathbf{r}_g = \left[x_g, y_g, z_g \right] \tag{3.9}$$

Now according to Fossen model, obtained rigid body inertia matrix is given as:

$$M_{rbm} = \begin{bmatrix} mI_{3\times3} & 0 \\ 0 & I_r \end{bmatrix} \tag{3.10}$$

Where *m* is the mass and I_r is the inertia tensor and it is defined as:

$$I_{\mathrm{r}} = \left[I_z \right] \tag{3.11}$$

And using Equation (3.10) and (3.11), we can obtain:

$$M_{rbm} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & -mr_g \\ 0 & 0 & m & 0 \\ 0 & -mr_g & 0 & I_z \end{bmatrix} \tag{3.12}$$

When a robot is model within a non-inertial frame, such as a rigid body frame so it experiences extra forces like Coriolis and centrifugal forces. The matrix below represents the force experience on the robot according to fossen model:

3.4.2 Hydrodynamics Equation

As water have high density compared to other means where robot and vehicles operate, so the drag component is greater. High speed is not required in robots but the drag component cannot be neglected for correct modelling. The drag force F_d can be given as:

$$F_d = \frac{1}{2} C_d \rho A v^2 \tag{3.13}$$

where F_d is the drag coefficient, A represent the reference area and v^2 is the velocity vector. As we had made an assumption of low velocity so this quadratic approximation can be used:

$$D(\nu) = D_{ld} + D_{nld}(\nu) \tag{3.14}$$

where the linear vector D_{ld} represent the damping force acting linearly along with velocity in each degree of freedom and it can be represented as,

$$= \begin{bmatrix} -X_u & 0 & 0 & 0 \\ 0 & -Y_v & 0 & 0 \\ 0 & 0 & -Z_w & 0 \\ 0 & 0 & 0 & -N_r \end{bmatrix} \tag{3.15}$$

and nonlinear damping vector D_{nld} consist of velocity dependent damping forces, which depend on the magnitude of the velocity in each direction and it can be expressed as.

$$= \begin{bmatrix} -X_{u|u}|u| & 0 & 0 & 0 \\ 0 & -Y_{v|v}|v| & 0 & 0 \\ 0 & 0 & -Z_{w|w}|w| & 0 \\ 0 & 0 & 0 & -N_{r|r}|r| \end{bmatrix} \tag{3.16}$$

3.4.3 Hydro-static Equation

Considering the mass m of robot measured in kg, acceleration g due to gravity, water density ρ measured in kg/m* and the volume of fluid as V measured in m*. Therefore the weight of robot is expressed as $W = mg$ and the buoyancy force is expressed as $B_f = \rho g V$. As the robot is in the Z plane so the position of robot can be expressed as $\mathbf{r}_p = [0, 0, z_p]^T$ and the overall restoring force $g(\eta)$ for the robot can be calculated as:

$$g(\eta) = \begin{bmatrix} (W - B_f)\sin\theta \\ -(W - B_f)\cos\theta\sin\phi \\ -(W - B_f)\cos\theta\cos\phi \\ z_p W\cos\theta\sin\phi \\ z_p W\sin\theta \\ 0 \end{bmatrix} \tag{3.17}$$

3.4.4 External Forces

To model the external forces, produce during the motion of an underwater robot are also due to propellers. Typically, two propellers are used for surge and sway forces, one for vertical motion and two for yaw moments. The thrust and moment produce by the propellers can be model using the control input vector and the thrust vector which is given as:

$$F_t = \tau = \left[t_1, t_2, t_3, t_4, t_5 \right] \tag{3.18}$$

where t_i shows the thrust produced by each propeller and is further calculated using the control allocation matrix **A**:

$$\tau = Au \tag{3.19}$$

The **A** matrix for the system of 4 propellers is given as:

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{3.20}$$

3.4.4 Dynamic Model

In summary, for achieving precise position and motion control the complete dynamic model can be achieve using equation (3.5) and (3.7) where:

$$M = M_{rbm} + M_{amm} \tag{3.21}$$

$$C = C_{rbm} + C_{amm} \tag{3.22}$$

The dynamic model can be expressed as above but here we are representing in the non- linear model form. The dynamic equation is given by:

$$M\dot{\nu} = \tau + d_{ext} - C(\nu)\nu - D(\nu)\nu - g(\eta) \tag{3.23}$$

or we can rewrite the above equation as:

$$\dot{\nu} = M^{-1}\left[\tau + d_{ext} - C(\nu)\nu - D(\nu)\nu - g(\eta)\right] \tag{3.24}$$

In state space ν is the state variable vector which include velocities and other dynamics of the system including the external disturbances caused during the movement in the aquatic environment.

3.5 Model for Position Control Design

For low speed motion we have made more assumption: vehicle has three planes of symmetry, mass distribution is homogeneous [14]. As a result, for the position control we are focusing on surge, sway and yaw while neglecting the z-direction to make controller more capable of capturing the essential dynamics required for effective position control. Now the inertia matrix becomes:

$$M = \begin{bmatrix} M_u & 0 & 0 \\ 0 & M_v & 0 \\ 0 & 0 & M_r \end{bmatrix}, \qquad (3.25)$$

where M_u, M_v and M_r is the difference between the inertia mass and added mass. Effect of restoring force is neglected because its only effect vertical dynamics like heave so now damping matrix become:

$$D(v) = \begin{bmatrix} X_{u|u}|u| & 0 & 0 \\ 0 & Y_{v|v}|v| & 0 \\ 0 & 0 & N_{r|r}|r| \end{bmatrix}, \tag{3.26}$$

The Coriolis and centripetal matrix becomes:

$$C(v) = \begin{bmatrix} 0 & 0 & -M_v v \\ 0 & 0 & M_u u \\ M_v v - M_u u & 0 & 0 \end{bmatrix}. \tag{3.27}$$

Now the external force acting on the robot is given as:

$$\tau = [F_u, F_v, F_r]^T \tag{3.28}$$

and from the equation (3.7), (3.25), (3.26) and (3.27), we can obtain separate equations for surge, sway and yaw.

$$\dot{u} = \frac{M_v}{M_u}vr - \frac{X_u}{M_u}u - \frac{D_{u|u}}{M_u}u|u| + \frac{F_u}{M_u} \tag{3.29}$$

$$\dot{v} = -\frac{M_u}{M_v}ur - \frac{Y_v}{M_v}v - \frac{D_{v|v}}{M_v}v|v| + \frac{F_v}{M_v} \tag{3.30}$$

$$\dot{r} = \frac{M_u - M_v}{M_r}uv - \frac{N_r}{M_r}r - \frac{D_{r|r}}{M_r}r|r| + \frac{F_r}{M_r} \tag{3.31}$$

These equations present the fundamental dynamics of the underwater robot for the position control design. The resulting equations of motion provide a comprehensive description of hoe surge, sway and yaw evolve over the time in response to the external forces and moments.

Now, considering the dynamic model as derived above in (3.24):

$$\dot{\nu} = M^{-1}\left[\tau + d_{ext} - C(\nu)\nu - D(\nu)\nu - g(\eta)\right] \tag{3.32}$$

The kinematic equation is simplified by using rotational matrix:

$$R_z(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.33}$$

And the kinematic simplified equation can give as:

$$\dot{\eta} = R_z(\psi)\nu$$

$$= \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{3.34}$$

Further expanding the above kinematic equations:

$$\dot{x} = u\cos\psi - v\sin\psi, \tag{3.35}$$

$$\dot{y} = u\sin\psi + v\cos\psi, \tag{3.36}$$

$$\dot{\psi} = r. \tag{3.37}$$

In summary, achieving position control of underwater robot, the system state is defined as:

$$\dot{x} = \begin{bmatrix} \dot{\eta} \\ \dot{v} \end{bmatrix}$$

$$= \begin{bmatrix} R_z(\psi)\nu \\ M^{-1}\left[\tau + d_{ext} - C(\nu)\nu - D(\nu)\nu - g(\eta)\right] \end{bmatrix} \tag{3.38}$$

This equation provides an expression of motion specially how the position of the robot relates to its velocity in a transformed coordinate and how dynamics of the system is influenced by the external disturbances including thrust, Coriolis effect and various other forces.

Where the state vector *x* is made up of velocity and motion which is given as:

$$x = \begin{bmatrix} x, y, \psi, u, v, r \end{bmatrix} \tag{3.39}$$

and the control vector *u* is consisting of the force generated by thrusters and given as:

$$u_f = \begin{bmatrix} u_1, u_2, u_3, u_4, u_5 \end{bmatrix} \tag{3.40}$$

3.6 Summary

This chapter have provided a framework of modelling for the dynamics of underwater position and motion control concentrating on its kinematic and kinetic characteristics. By utilizing Fossen theory, we derived the essential equations that describe the robot motion by integrating the rigid body dynamics, external disturbances and other static and dynamics aspects as well. By obtaining linear and nonlinear characteristics of the system, we aim to enhance the accuracy and robustness for the position control solutions that will be discussed in next chapters. Overall this foundational research not only deepen our understanding of underwater robotics but also enhance the advancement in autonomous underwater robotics technology.

Moreover, control of underwater robot is a challenge due to nonlinear velocity, hydrodynamic forces that are acting on the robot. Moreover, the underwater robot mostly operates autonomously and carry a very sensitive and expensive equipment so a better and precise control system is requiring for precise control. The dynamic model and its parameters also effect the controller performance that why it is necessary to put effort in estimation and validation of model parameter. Dynamic model overview is commonly useful for underwater robots as the Fossen model for marine craft is very well suited for modelling the underwater robot. Since the main purpose is the control of robot so this modelling was done. This is for the better understanding of the robot dynamics with in the given axes.

CHAPTER NO. 04: CONTROL SYSTEM DESIGN

4.1 Background

The control system plays a very crucial role in obtaining the desire Behaviour of the system, particularly in dynamic environment where stability, accuracy, precision and robustness is very essential. Control system designing involves designing a controller that operates the system to achieve specific goals, such as maintaining stability, optimising performances and tracking a reference trajectory etc. over time control system has evolved from simple and classical methods to more advance and adaptive methods with capabilities of handling more complex and multivariable systems. This evolution has become beneficial in areas likes aerospace, robotics and underwater applications where environment is highly unpredictable with nonlinear dynamic and subject to many other external disturbances [8].

One of the earliest approaches to control system design were PID controller, which became very significant importance in the field of control system because of its simplicity and effectiveness in managing linear system. However, the growing demand for better performance in nonlinear and time varying systems cannot be obtaining with PID controllers. Because this method cannot handle the multiple inputs, states and uncertainties in the model dynamics [15]. These limitations help in the development of better and sophisticated approaches like state space control which offer better flexibility and performance in modern application.

In recent years, MPC has gained significant importance in control system theory for controlling multivariable and nonlinear system with high precision [10,11]. MPC operates by predicting future behaviour of the model, optimising control input over the prediction horizon and by applying optimal control action at each step. This approach allows MPC to handle variable explicitly and making it well suited for complex nonlinear systems [2].

Now the feedback mechanism is use in robot system to maintain the desire position through the thrusters. The desire position of the robot is given as: $\eta_p = [x_p, y_p, \psi_p]$ and the cost function J of MPC is given as:

$$J = \int_0^T \left(\tilde{x}(s)^T Q \tilde{x}(s) + \hat{u_f}(s)^T R \hat{u_f}(s)\right) ds + \tilde{x}(T)^T P \tilde{x}(T) \tag{4.1}$$

$$\begin{aligned} \text{minimize} \quad & J \\ \text{subject to} \quad & \tilde{x}(s) = f(\hat{x}(s), \hat{u_f}(s)), \\ & \hat{x}(0) = x(t_0), \\ & |\hat{u_f}(s)| \leq u_{\text{fmax}}, \quad \forall s \in [0, T]. \end{aligned}$$

Where $\tilde{x}(s)$ represents predicted state trajectory, $\hat{u}(s)$ represent the set of control input,

Q, P and R are the weighting matrices and $\hat{x}(0)$ represent state error. To address the stability concerns in underwater robots, control method like MPC have been used very vastly. But standard MPC does not provide precise stability especially where continuously unpredictable disturbances are occurring. So an advance MPC must be designed to ensure system remain stable throughout. Several types of MPC were developed to address specific challenges [4]. Some of widely used are given below:

- **Standard Model Predictive Control (SMPC)** optimise the control input over the prediction horizon without explicitly ensuring stability. SMPC predict future state based on the system dynamics. It doesn't guarantee stability until additional con- straints are not added to the cost function. SMPC can result in instability if it is not fine-tuned.
- **Robust Model Predictive Control (RMPC)** is designed to handle uncertainties and disturbances. RMPC is used to ensure that control performance is maintained even the system dynamics are not perfectly known. But here we have done detail modelling of the robot dynamics so this can be used but it will be better to use other strategy that offer better control over known dynamics. On the other hand, it requires solving more complex optimization problem and while dealing with all uncertainties, the control action may be more conservative and cause low performance.
- **Linear Quadratic Regulator Model Predictive Control (LQR MPC)** is an optimal control method that minimize the cost function of state and control input. LQR provide an optimal solution and easy to implement and analyse but as mentioned in name its linear quadratic regulator so it is most widely used in linear system with known dynamics. It performance degrades in the presence of significant nonlinearities.
- **Lyapunov Model Predictive Control (LMPC)** is a control strategy that combines MPC with lyapunov stability principal to ensure system stability while performing predictive control. A lyapunov function is define to ensure system stability. Lyapunov function is

incorporated with MPC optimization problem which ensure each control action to reduce the lyapunov function value over the time which helps to minimizing cost function and ensuring stability. LMPC handle uncertainties and disturbances more than traditional MPC and handle the state and input effectively to provide an optimal solution. While designing lyapunov function for a complex system, it requires complete system dynamics that we have already achieve in the previous chapter [4, 7].

Among the all above methods, LMPC is an attractive choice where stability and precision is main concern especially in a system with nonlinearities or external disturbances. LMPC ensure stability using the lyapunov function. This is particularly beneficial in application like underwater robotics, aerospace where unpredictable dynamics and environmental disturbances are occurring. By selecting LMPC, control design utilizes full capabilities of MPC while ensuring stability in presence of disturbance and modelling inaccuracies. This make attractive and superior choice as it offers a precision, stability, robustness over other control strategy of MPC.

4.2 Control Development

Here we are focusing on the design of control system for the model developed in previous chapter for the dynamic position control of the underwater robot which is expressed as:

$$\dot{x} = f(x, u_f) = \begin{bmatrix} R_z(\psi)\nu \\ \dot{\nu} = M^{-1}\left[\tau + d_{ext} - C(\nu)\nu - D(\nu)\nu - g(\eta)\right] \end{bmatrix} \tag{4.2}$$

As shown in the above figure, control loop of MPC receive reference input with error and after optimising it with prediction model produce an output for the system.

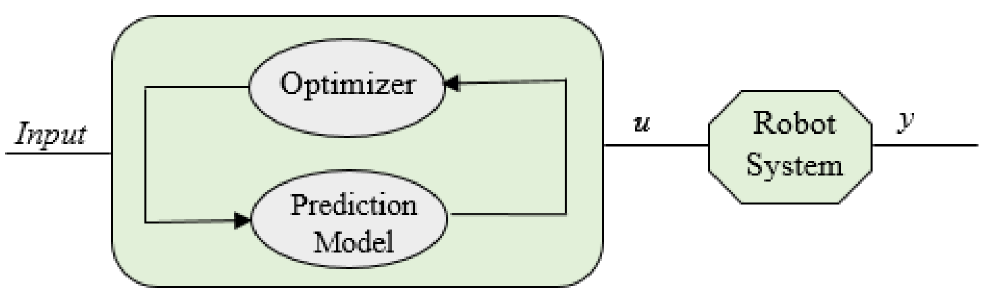

Figure 2: Control loop of MPC

However, if we find best control solution using cost function equation (4.1), it is not guaranteed that the robot will remain stable. So to ensure good performance, we need to design the control strategy in advance for the complex systems. For this we must develop additional feedback control. As discussed above lyapunov method is more reliable and use widely in obtaining stability of the nonlinear system. So now redefining the cost function with addition of lyapunov function:

$$J = \int_0^T \left(\tilde{x}(s)^T Q \tilde{x}(s) + \hat{u_f}(s)^T R \hat{u_f}(s)\right) ds + \tilde{x}(T)^T P \tilde{x}(T) \tag{4.3}$$

$$\begin{aligned} \text{minimize} \quad & J \\ \text{subject to} \quad & \tilde{x}(s) = f(\hat{x}(s), \hat{u_f}(s)), \\ & \hat{x}(0) = x(t_0), \\ & |\hat{u_f}(s)| \leq u_{\max}, \quad \forall s \in [0, T]. \end{aligned}$$

$$\frac{\partial V}{\partial x} f(\hat{x}(0), \hat{u_f}(0)) \leq \frac{\partial V}{\partial x} f(\hat{x}(0), h(\hat{x}(0)))$$

V is a scalar function use to prove stability of dynamics system and $h(x\hat{}(0)$ represent the existing controller, which is a function of state variable and it provide output as control action based on current state.

The LMPC algorithm update control input based on the current state and solve a finite horizon optimization problem while ensuring stability through the Lyapunov function [19]. Its allows the system to overcome the unpredictable changes in a dynamic environment and make the system effective. This enables the controller to adapt the changes in the system dynamics while maintaining desire position and stability.

4.2.1 Lyapunov function

Since we are using Lyapunov MPC control for nonlinear dynamics so in this section we are modelling Lyapunov function to attain stability and robust control. The Lyapunov based MPC is widely used for the dynamic position control and by utilizing a nonlinear controller with Lyapunov function guarantee the desire position and system stability. By constructing a control system within Lyapunov framework, we aim to maintain stability and effectiveness of the system to achieve desire position.

The Lyapunov function can be defined as measure of energy in the systems, typically kinetic and potential energy. kinetic energy is associated with motion of robot and the potential energy is associated with position of robot [20].

From the equation (3.23) and (3.34), the kinetic energy of the system is related to velocity and inertia matrix. Thus kinetic energy is represented as $\frac{1}{2} v^T Mv$. This term measure the energy store in the system due to velocity, since this equation become part of Lyapunov function:

$$V_1(v) = \frac{1}{2} v^T M v \tag{4.4}$$

Now let include potential energy associated with position. The η represents the gravitational force. The position error is often represented as $\tilde{\eta} = \eta - \eta_d$ and the standard of potential energy term represented as:

$$V_2(\eta) = \frac{1}{2} \tilde{\eta}^T K_p \tilde{\eta} \tag{4.5}$$

Considering the above equation (4.4) and (4.5), Now let combine the both term, potential energy due to position error and kinetic energy due to velocity and the equation become:

$$V = V(v, \tilde{\eta}) = \frac{1}{2} v^T M v + \frac{1}{2} \tilde{\eta}^T K_p \tilde{\eta} \tag{4.6}$$

where V is the velocity vector, $\tilde{\eta}$ is the position error and K_p is the diagonal gain rep- resenting position control gain. Now taking the derivative of V along the trajectory of the closed loop system:

$$\dot{V} = v^T M \dot{v} + \dot{\eta}^T K_p \tilde{\eta} \tag{4.7}$$

and from equation (3.34) substituting $\dot{\eta} = R(\psi)v$,

$$\dot{V} = v^T M \dot{v} + v^T R^T(\psi) K_p \tilde{\eta} \tag{4.8}$$

Now, substituting equation (3.23) into above equation:

$$\dot{V} = v^T [\tau + d_{est} - C(v)v - D(v)v - g(\eta)] + v^T R^T(\psi) K_p \tilde{\eta} \tag{4.9}$$

Now for the mathematical simplification an assumption of $C(v)v = 0$ is made because it does not contribute to $\dot{V}$ for specific state also it is dependent on velocity so velocity at low velocity we can model it as zero and the above equation is given as:

$$\dot{V} = v^T [\tau + d_{est} - D(v)v - g(\eta)] + v^T R^T(\psi) K_p \tilde{\eta} \tag{4.10}$$

The first part of the above equation is indicating how control input and dynamics influence the energy of the system while negative value suggest the system is stabilizing and positive indicate instability. The second part indicate the control gain contribution in correcting position error while affecting the overall stability and now taking the velocity transpose matrix common, above equation become:

$$\dot{V} = v^T [\tau + d_{est} - D(v)v - g(\eta) + R^T(\psi) K_p \tilde{\eta}] \tag{4.11}$$

Overall this equation is the critical component in analysing and ensuring the stability in the control system. Its provide the information of the control input, system dynamics and how effectively the control system reduce the position error to stabilize the system.

4.3 Stability Analysis

Stability analysis is very important in control theory and dynamical systems as it help to determine the system Behaviour how its response to the disturbances. Understanding stability involves various factors as after some disturbances a system return to its equilibrium state is considered stable while unstable system diverges from the desire position/state. By using lyapunov’s method, which is dependent on Lyapunov function, we can evaluate the system stability using the below methodologies:

4.3.1 Time Derivative of Lyapunov Function:

The stability is analysed by obtaining derivative of Lyapunov function, denoted as $\dot{V}$. If $\dot{V}$ is negative ($\dot{V} < 0$), then this mean that the energy of this system is not increasing which mean that the system is marginally stable. The energy may decrease continuously to attain equilibrium and reaching ($\dot{V} = 0$), which indicate that the system is stable and it will not oscillate more.

4.3.2 Negative Definiteness and Asymptotic stability:

If $\dot{V}$ is negative define i.e. $\dot{V} < 0$ for all $x = 0$, it represents that the energy of system is steadily decreasing over time which indicate that the system state is moving closer to equilibrium point and confirms that the system is asymptotically stable. Asymptotically stable means if any change occur due to small disturbances then the system will gradually return to its equilibrium point.

4.3.3 LaSalle’s In-variance Principle:

This theorem provides a more understanding of stability based on lyapunov function. According to this theorem, if $\dot{V}$ is negative semi definite then the system eventually settles into the condition $\dot{V} = 0$. This mean that the system does not experience any further disturbances and it will settle as stable configuration. Stability analysis using the Lyapunov method is a promising approach for attaining stability. A negative time derivative of the function indicate that system energy is decreasing or remaining constant, suggesting the system is stable. Through this framework, engineer and scientist can design a better control system which they can effectively manage and stabilize the complex nonlinear systems [8].

4.4 Summary

In this chapter, we developed the LMPC framework for the position control of the under- water robot. The model developed in previous chapter incorporates kinematic and dynamic equation which serve as foundation for the position control. The LMPC here is designed to optimise the problem constraints on the control input and a prediction horizon to maintain the desired position and orientation. A lyapunov function is especially designed for the stability. This approach allows the robot to maintain stability for the desire equilibrium point.

In summary, this controller offers a robust solution for the positioning control that is combined with the cost function and lyaunov stability function. The resultant control strategy is capable of maintaining the position in an improved, responsive and effective way.

CHAPTER NO. 05: SIMULATIONS AND RESULTS

5.1 Introduction

This chapter discuss the simulation outcomes for the underwater robot system. The simulation is designed to evaluate the MPC performance in achieving the desire position control, handling disturbances and managing system constraints in real time. Each section will pro- vide demonstration of the simulation setup, key parameters, test scenarios and performance metrics. The results are used to verify the systems response and validate the MPC design choices discussed in the prior chapter.

5.2 Simulation Setup and Parameters

5.2.1 Overview of the System Model

The system model implemented in the simulation is derived from the mathematical model developed in the chapter 3. The developed model describes the robot dynamics, including state variable, system inputs, system outputs, disturbances and other forces affecting position control. The LMPC is designed on these dynamics to maintain precise position within the set boundary despite possible disturbances.

5.2.2 LMPC Parameter

Simulation of LMPC based controller is tested on the model developed through a reference trajectory which validate the controller capabilities to mitigate the disturbances and accurately track the trajectory. Key parameters are tuned for the optimal control, including the prediction horizon set to 20. Weighting matrix is important for minimizing the tracking error. To prioritize stability and fast response the weighting matrix is given as:

$$Q = \mathrm{diag}(10^6, 10^6, 10^5, 10^3, 10^3, 10^3)$$

The control on input matrix to avoid excessive input energy and to ensure balance control is given by weighting matrix *R*:

$$R = diag(10^{-4}, 10^{-4}, 10^{-4}, 10^{-4})$$

Where the weighting matrix P give the terminal state cost and should be developed precisely to ensure stability at predication horizon and it is given as:

$$P = diag(10^{4}, 10^{4}, 10^{3}, 10, 10, 10)$$

To avoid ambiguity in which kind of control parameters are used in the system, further information concerning the Model Predictive Control (MPC) architecture was incorporated. We decided to set the prediction horizon equal to 20 for further computation optimization while achieving fairly accurate performance. With this, weighting matrices were developed to focus on how precise the tracking must be and energy optimization control. Namely, the state weighting matrix (Q) was selected to balance the control effort of the system. (R) also made provision for a balanced use of energy. Further, the terminal cost weighting matrix (P) equipped the control law with the capability of tracking the desired values maintained for stability at the prediction horizon. Saturated constraints were used to contain the control signals into operation safe ranges for the sake of the thrusters. To achieve the reproducibility and offer additional implementation details, the MATLAB codes for the development of MPC framework along with the tuning parameters of the proposed work and the scripts used for the real-time simulations are provided in the Appendix section. These points make the work transparent, and also create a road map for other researchers to follow.

5.2.3 Robot Parameter

In order to make the impact of system implementation more realistic, such main physical performance factors of the underwater robot as total mass, maximum velocity and maneuverability were included into the model. The weight of the robot is about 72.6 kg, that allows the robot to work safely in the water up to 12 meters depth, and with special rating up to 15 meters. It possesses 5 propellers; pitching, surging, swaying, heaving, yawing movements of the AUV are controlled and it is made to work at low velocities in order to reduce drag forces. Moreover, some white noise for identifying measurements errors and external range of interference were also applied. The noise in measured quantities was assumed to be Gaussian with standard deviation of 0.05 m/s for the velocity and 0.01 m for position sensors.

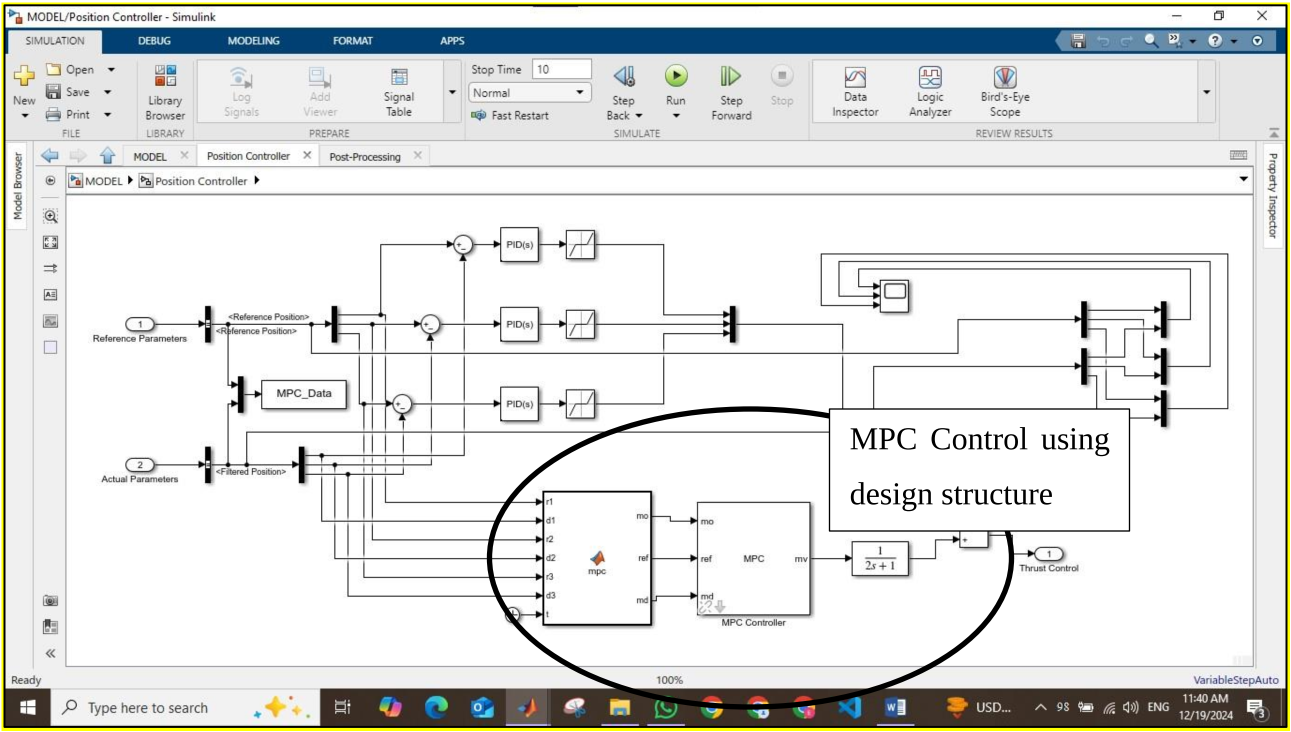

Figure 3: Reference parameters for robot control in Simulink

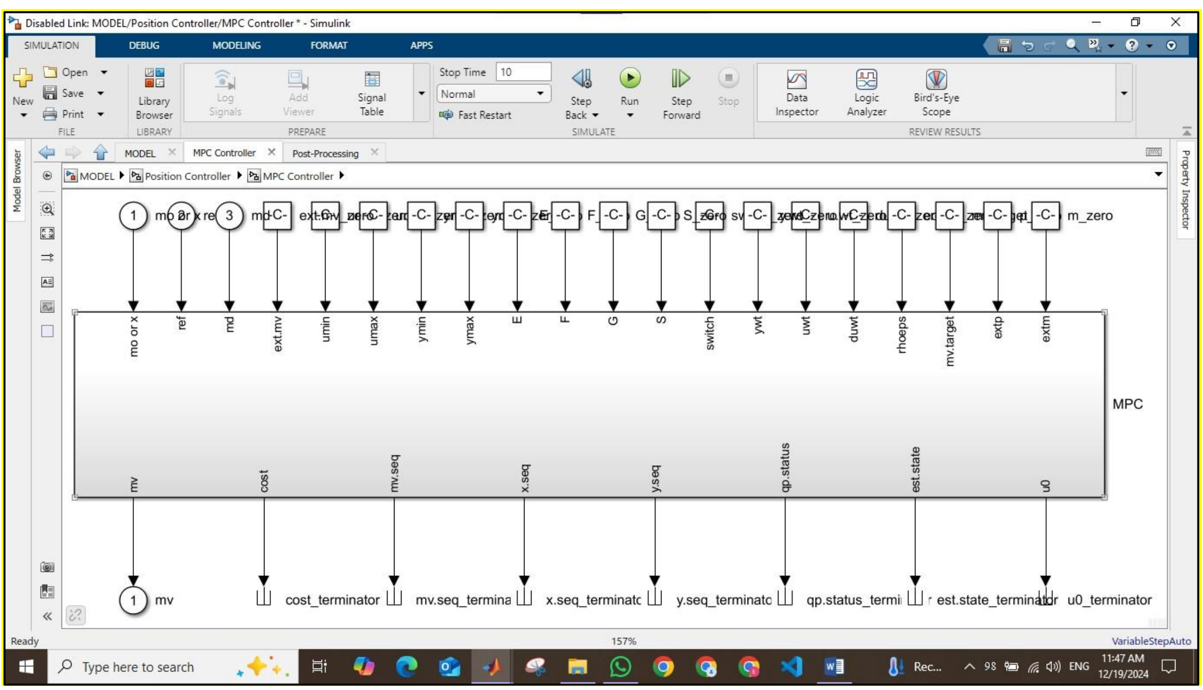

Figure 4: MPC Control parameters for Robot

These both figures 3 and 4, illustrates the arrangement of reference parameters that enables the control of the underwater robot in Simulink. The reference parameters are the control signals to the control system given that they describe the desired path and performance standards of the robot. The figure enlarges the positional and velocity reference inputs which are important for the accurate tracking of the robot path. All of these parameters are directly fed into the Model Predictive Controller (MPC) where the control actions to keep the robot on track to follow the desired trajectory are calculated. When substituted for these reference parameters, the simulation guarantees uniformity in measuring the controller's performance in various conditions. MPC control parameters set for the robot and includes prediction and control horizons with weighing matrix of states and control inputs. These parameters are relevant when it comes to the optimization of the control variables as used in the MPC. The prediction horizon reflects how many steps in the future the controller plans for the robots action and the control horizon reflects how many of the control actions will be optimized. The weighting matrices are developed to compromise between trajectory tracking error and control energy to make sure that the controller runs effectively without overloading the actuators. It is essential for stability and reach viable performance under changed environment and conditions in this configuration.

Uniform forces paralleling specific axes to emulate external interferences such as ocean currents while sinusoidal force perturbation with distinct frequencies were used to mimic time variant disturbances. These coming augmentations give an appearance of a more realistic overall depiction of the system, offering an accurate validation within dynamic and noised contexts for control methods.

5.2.3.1 Code Snippet

The MATLAB function mpc implements a Model Predictive Control (MPC) framework for three control variables (u1, u2, and u3) based on reference (r1, r2,r3) and disturbance (d1,d2,d3) inputs over a specified time vector (t) that are given in the appendix section. It begins by defining control parameters, including time step delta, prediction horizon N, and weighting matrices Q, R, and P for state and control penalties. PID gains (Kp, Ki, Kd) are set for each control variable to address proportional, integral, and derivative errors.

Control actions are calculated iteratively over time using the PID formula, with integral and derivative terms updated at each time step. The control inputs are optimized at each step using MPC by solving a quadratic programming problem, ensuring the system operates efficiently under the defined constraints. Finally, the system's state and control actions are used to compute a Lyapunov function value as a stability measure. Key variables like M, the system mass matrix, and Kp, the gain matrix, are employed in the Lyapunov function ensuring stability across iterations.

5.2.4 Simulation Results

5.2.4.1 Noise-Free Simulation

The simulation results obtained from the modelling and simulation of robot position control system using a Lyapunov Model Predictive Controller (LMPC) is given below. The initial condition for the trajectory were set at $x(i) = [0, 0, 0, 0, 0, 0]$, with mid interval at $x(m) = [-8, -8, pi, 0, 0, 0]$ and the final trajectory is also set at $x(f) = [0, 0, 0, 0, 0, 0]$, which is actually representing the robot's start and end point.

Here, we are analysing the robot's trajectory in three dimensional and two dimensional space, along with the individual state response of the surge, sway and yaw and checking the control ability of the controller along multiple conditions.

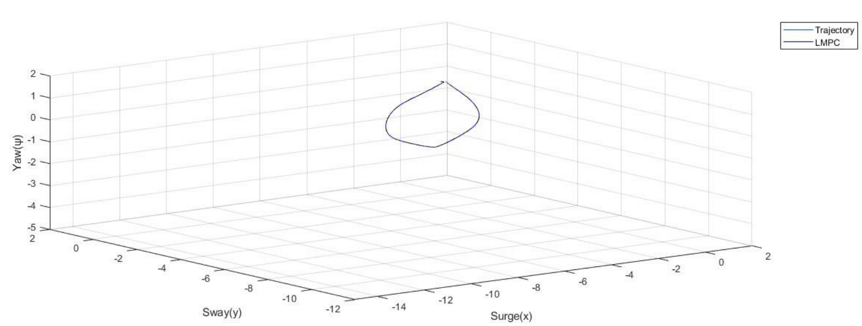

Figure 5: Robot trajectory in 3 dimension

The figure 5 beaches the robot's path in a 3-D environment in noiseless circumstance where surge (x), sway (y), and yaw (ψ) movements are visualized. The path indicates how the robot responds to the reference path that has been set and established the performance of the Lyapunov-based Model Predictive Controller (LMPC). The actual path is depicted by the continuous line since it is close to the desired path hence predicting less positional mistake. The controller valuable and efficiency is underscored by the absence of disturbances which makes the simulation perfect for real-time underwater navigation.

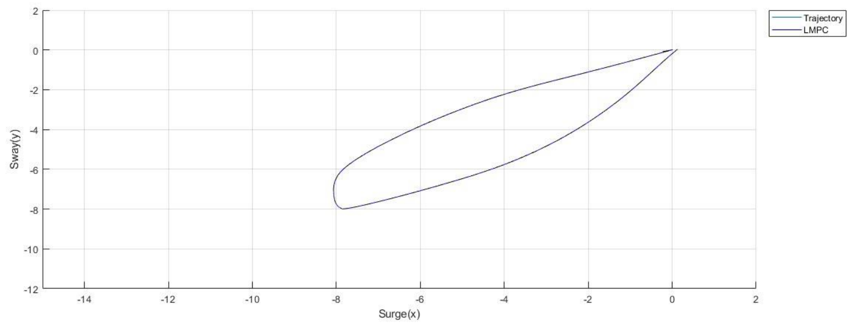

Figure 6: Robot trajectory in Local Plane

The figure 6 represents a 2D planar view of the robot's path planning from the top view, specifically emphasizing the swaying motion (x) and surge (y) positioning on the X (interpolated) and y axes only in a noiseless condition. This was done while demonstrating maneuvering on the horizontal plane with high accuracy since the actual robot path closely follows the intended path. The controller also reciprocates a mastery of transition between two waypoints and keeps an adequate control to prevent deviations: this presents the position control ability. This view is particularly advantageous for exploring the behavior of the robot in applications involving traveling in a lateral plane, e.g., offshore surveys, pipeline inspections.

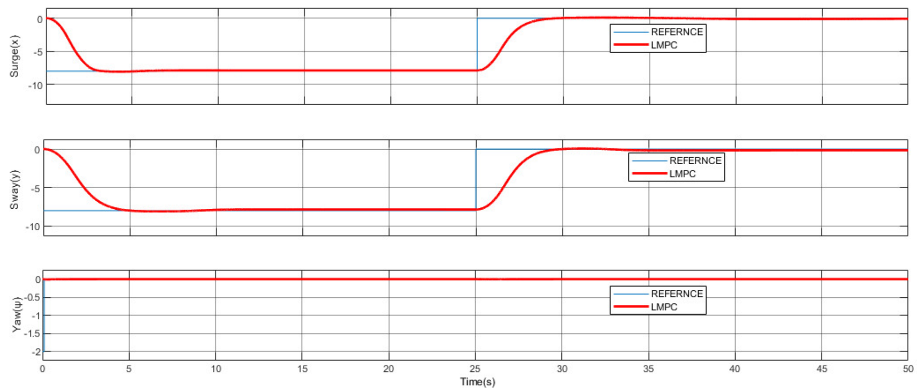

Figure 7: Disturbances estimation of constant currents

This figure includes three subplots of the robot's response to the current disturbances in surge, sway, and yaw that are present continually. The first plot illustrates the temporal behavior of the surge response where the controller counteracts these positional deviations due to disturbances. The second plot represents sway stabilization proving that controlling of side movements is also a strong side of the system. The third plot relates to yaw, in which the controller stays aligned even when forces are applied from the outside. Collectively, these plots confirm the effectiveness of the LMPC in rejecting steady disturbance while keeping the robot on the correct trajectory.

The simulation results demonstrate that the LMPC is highly effective at controlling the robot position and orientation. The controller successfully follows the initial trajectory in three dimension and reaches the final to the final trajectory that is predefined. These plots collectively represent the robustness of the LMPC in managing the robot's trajectory and orientation, ensuring in high precision across the reference trajectory.

5.2.4.2 Simulation with Noise

Let's observe the robots position control in noise and disturbances achieved through the LMPC controller. In this simulation a model noise is induced and making the control sys- tem more complex and challenging. The initial state vector is kept same $x\,(0) = [0, 0, 0, 0, 0, 0]$ with mid interval at $x(m) = [-8, -8, pi, 0, 0, 0]$ and the final trajectory is also set at $x(f\,) = [0, 0, 0, 0, 0, 0]$. Despite the noise, LMPC aims to follow the position trajectory and orientation.

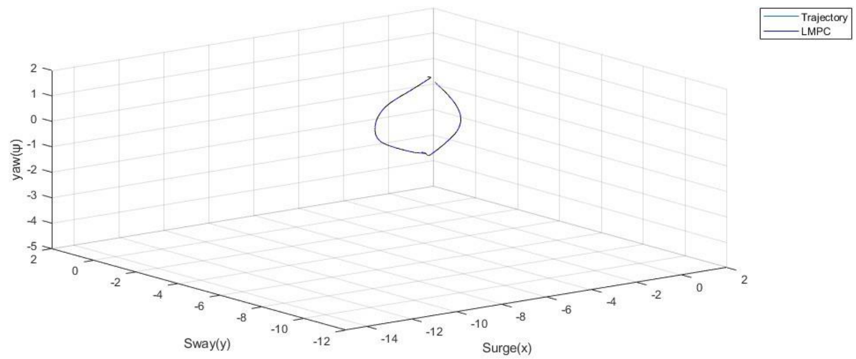

Figure 8: Robot trajectory in 3 dimension with noise

This figure illustrates the robot's motion in terms of 3D path trajectories based on noisy scenarios of application such as varying ocean currents. Despite the noise, the LMPC keeps the position of the robot in the proximity to the desired path and small oscillations are detectable along the path. Such deviations happen because the environment is constantly changing, but the controller succeeds in mitigating these effects to enable the robot to learn without drifting off too much. This performance shows how the controller is effective at stabilizing the operation in the face of uncertainty that is common when the operation is taking place underwater.

This 2D trajectory plot shows the horizontal position of the robot in the sway and surge form. Due to noise the path face minor fluctuations and when the robot reaches the desire position LMPC try to adjust the position and keep the position in the desire trajectory with small deviation and does not allow the disturbances to affect and overcome on the controller design to deviate completely.

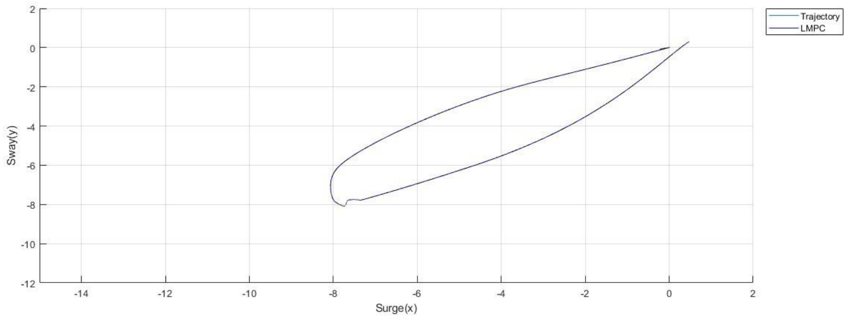

Figure 9: Robot trajectory in Local Plane

This figure gives a plan view of how the robot moves over ground in the xy-axis in the noisy condition with respect to sway (x) and surge (y) axes. Though there are some deviation on the graph due to noise the system plot almost parallel to the set path which is the measured capability of LMPC. It actively translates the observed offset from the reference trajectory and directs the dynamic change in robotic movements. Such behavior reflects that the controller is accurate in horizontal maneuvers, including when in a complex environment.

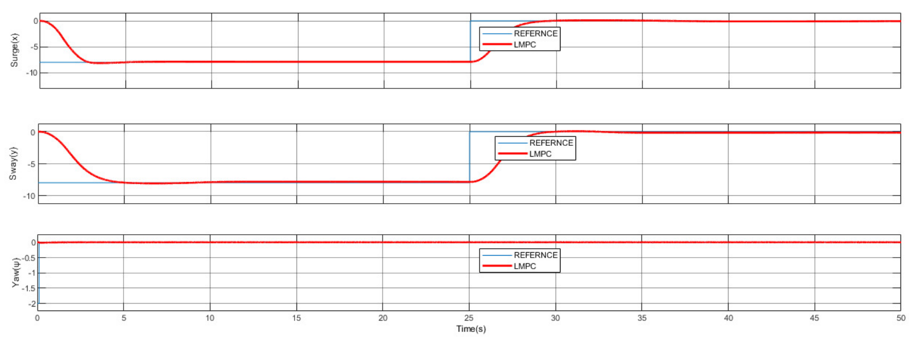

Figure 10: Disturbances estimation of Noisy currents

This figure shows the robot’s performance in terms of surge, sway and yaw noise disturbance in time response. The first plot illustrates the surge response in which the controller rapidly dampens oscillations and navigates the robot in the correct path. The middle plot depicts swaying which shows how the systems can be restored to their nominal position in case of lateral shocks by noise.

The lower plot represents the yaw response; it is seen that the controller helps to restore the orientation of the robot even when external disturbance affects it. These results indicate resilience of the LMPC in minimizing the noises interference while enhancing a strict control over the robots path and orientation.

The simulation results is slightly different from the above analysis in which no noise were consider but this demonstrate the LMPC ability and controller robustness to overcome the noise and keep the trajectory very near to the desire position. The individual, three dimensional and two dimensional shows the impact of the noise on each variable and how it is affecting the surge, sway and yaw.

5.3 Summary

The results from these simulation shows the effectiveness of LMPC in managing robot position and orientation. The 2D and 3D trajectory illustrate the noise free and with noise effect on the controller, keeping the robot close to the reference position. The individual state response also reveals the controller performance in the dynamic environment. In summary the control simulation was conducted to evaluate the performance, stability of the controller for different scenarios. The results obtain provide an overview of a robot position control system through the control of surge, sway and yaw. The ability to maintain precise position control is an essential strategy for the growing real world application in the robotics field.

CHAPTER NO. 06: CONCLUSION

6.1 Conclusion

This thesis explored the dynamic position control of underwater robot using Lyapunov based model predictive control. With the rising demand for autonomous underwater robot in- creases, achieving accurate control has become more essential than ever. Our research focused to develop a control strategy that not only guarantees stability and performance but also respond effectively to the unpredictable challenges associated with underwater dynamics.

6.1.1 Summary of Research Objectives

The primary objective of this study were to design a robust control framework for underwater robotics using Lyapunov based Model Predictive Control to improve its effectiveness in maintaining accurate and precise position. This involve addressing many complexities of underwater dynamics, including buoyancy, hydrodynamic forces and disturbances caused by ocean currents. By utilizing Lyapunov functions we aimed to lay a solid foundation for ensuring both stability and performance in our control strategy. In addition, our research aims to enhance the adaptability of underwater robots, allowing them to more responsive and intelligent in dynamic unpredictable environment.

6.1.2 Key Findings

Our research yield significant finding that contribute in the advancement of underwater robotics theory. The use of Lyapunov MPC demonstrated a remarkable improvement in position tracking accuracy while compared to the other traditional control methods. Further- more, the predictive nature of MPC allow the controller to detect the changes in underwater environment in advance and enable timely adjustment that improved the overall performance of the system.

By conducting a comprehensive stability analysis using Lyapunov function, it is demonstrated that the Lyapunov MPC framework maintain global asymptotic stability for the underwater robotics system that means this level of stability ensures that if any uncertain- ties and disturbances face then the system will attain its stability. The principle of Lyapunov stability offer a strong foundation for the effectiveness of our control design, ensuring bet- ter and effective performance even under varying and unpredictable nature of underwater environment.

6.1.3 Contributions to the Field

The study and research finding significantly contributes to the advancement of underwater robotics. By designing a robust Lyapunov MPC approach, we have improved the control strategies especially for the underwater robotics in a dynamics underwater environment. This study not only demonstrate the effectiveness of MPC in complex environment and challenging scenarios but also play a vital role in the stability analysis of the underwater robotics using Lyapunov function. Additionally, our study emphasizes the potential to enhance the autonomy and reliability of underwater robotics in various practical applications, including underwater exploration, marine search and environmental monitoring. The knowledge gained from this study serve as an important resource for engineer and researcher to create advance control systems for underwater robotics.

6.1.4 Implications for Practice

The practical implication of this research are significant for the industries that rely on underwater robotics for various underwater operations. For example, in field of marine biology, improved control strategies can enhance the efficiency and precision of data collection during underwater surveys. In the oil and gas industry, underwater robots equipped with robust control techniques can conduct more inspection and maintenance activities with high reliability and minimizing risks.

Moreover, the use of Lyapunov MPC addresses many dynamic challenges faced by underwater robots in underwater environment. It provider wider applications in areas such as search and rescue operation, underwater construction and monitoring of aquatic ecosystems. By ensuring stable and accurate control, this study creates many opportunities for more efficient and reliable underwater operations

6.1.5 Limitations of the Study

This study encountered few limitations. One of key challenge is high computation intensity of MPC optimization problem, especially when applied in real time scenarios. However, this research primarily focused on simulations through the use of Simulink for stability and simulation analysis. Although we utilized nonlinear dynamics in our modelling with all external disturbances and dynamics but in real world scenario, underwater dynamics presents more additional uncertainties and unpredictable disturbances that may not full encounter in the simulation may impact control system performance.

In summary, this thesis emphasizes the importance of Lyapunov based MPC as an effective and reliable approach for dynamic position control in underwater robotics. This finding from the research not only enhance the control framework of the underwater robots but also provide a promising solution in improving the reliability and efficiency of robots during operations. As underwater robots gain advancement, the necessity for the robust control strategies will be critical. This research establishes a fundamental step toward further exploration and innovation in the control mechanism of underwater robot dynamic system.

6.2 Future Work

This research work done in this thesis has laid a solid foundation in the field of underwater robotics. While this research work was focused on dynamic position control of underwater robot so there are few things remain for the future research and development. This chapter will outline the potential future work of this study:

6.2.1 Optimising Computational Challenges

The computational demand of MPC in real time application present various challenges for the practical implementations. Further research should be carried out to explore the optimization technique that reduce the computational burden of the MPC optimization problem. An efficient solver algorithms technique could be developing to enable real time implementation of the multiple types of MPC in various application in industrial as well as scientific and research purposes.

6.2.2 Integration of Learning Algorithms

Further integrating machine learning algorithm with Lyapunov MPC will further enhance the adaptability and performance of the control system. By applying reinforcement and adaptive control technique, future work will allow the system to learn optimal Behaviour over time due to evolvement in the control system techniques.

6.2.3 System Validation

To validate the theoretical advancement made in this thesis, future work should focus on the experimental validation of the proposed control strategies in real world underwater robotics scenarios. Filed test should be carried out with Lyapunov MPC as it will provide valuable insights into the practical challenges and effectiveness of the control technique develop.

6.2.4 Collaborative Control

As underwater robotics has become area of interest for researcher and industrialist, so further research should be carried out to develop a collaborative control strategy that allow the multiple robots to work together seamlessly while maintaining stability and performance.

As we continue to explore the complexities of the control system of robotics, it is essential to do collaboration between researchers, engineer and industrialist to address the challenges and master the technique of control in underwater robotics theory as well as in practical implementation. The journey of discovering is ongoing and everyone is encouraging to push the boundaries of what is possible in underwater exploration and robotics.

The final section provides suggestions for future work and analysis of the results obtained, which are considered together. In this unified section, the major findings of the study are provided as follows: The current research contributes a comprehensive Lyapunov-based Model Predictive Control (MPC) approach for underwater robots for closed-loop dynamic positioning control under various environmental conditions. The study reveals specific gains in tracking precision, steadiness, and reliability over conventional control techniques. Further, it provides directions for future work, including fine-tuning computational difficulties for real-time application, integrating more advanced machine learning for dynamism into the framework, and using experimental simulations in real-world contexts. Both of these conclusions highlight the applied value of the study and its implications for the future development of AUVs as a branch of underwater robotics.

Bibliography

[1] Meng L, Wang Y. Modelling and control of underwater robotic systems: A survey. Journal of Marine Science and Engineering. 2022;10(6):835.

[2] Liu W, Liu Y. Modelling and simulation of an underwater robot based on multibody dynamics. Journal of Marine Science and Engineering. 2020;8(8):563.

[3] Srikrishnan G, Kumar D. Modelling and control of underwater robotic systems using reinforcement learning. Journal of Robotics and Mechatronics. 2022;34.

[4] Azhagappan S, Iqbal M. Lyapunov-based control for dynamic positioning of marine ves- sels. IEEE Trans Control Syst Technol. 2020;28(5):1377-1385.

[5] Shi, Y., Shen, C. Wei, H. Zhang, K. Advanced Model Predictive Control for Autonomous Marine Vehicles. Advances in Industrial Control Series. Springer Cham, 2023.

[6] Liu Y, Zhang T. Stability analysis of a DP system using Lyapunov functions. IFAC-PapersOnLine. 2021;54(21):76-81.

[7] Huang Y, Yu H. Adaptive control for dynamic positioning of vessels with Lyapunov stability. Ocean Eng. 2019;177:147-157.

[8] Rojas A, Mascaro R. Dynamic positioning using Lyapunov-based controllers: A survey. Robotics and Autonomous Systems. 2021;131:103712.

[9] Goh K, Hu L. Dynamic positioning for marine operations: A comprehensive review. IEEE Trans Robotics. 2021;37(3):645-663.

[10] Naderpour M, Shamsollahi M. Model predictive control for dynamic positioning of ma- rine vessels. IEEE Trans Control Syst Technol. 2019;27(2):591-601.

[11] Wang J, Chen H. Robust control of dynamic positioning systems under environmental disturbances. Control Eng Pract. 2020;96:104166.

[12] Hussain M, Ahmad R. Dynamic positioning of marine vessels: A hybrid approach using fuzzy logic and neural networks. J Marine Sci Eng. 2021;9(5):507.

[13] Leong K, Wong D, Chan C. Dynamic positioning control using sliding mode control for marine vessels. Robotics. 2020;9(2):43.

[14] Hu Y, Li B, Jiang B, Han J, Wen C-Y. Disturbance observer-based model predictive control for an unmanned underwater vehicle. J Marine Sci Eng. 2024.

[15] Shi Y, Shen C, Wei H, Zhang K. Advanced model predictive control for autonomous marine vehicles. 1st ed. Cham, Switzerland: Springer; 2023.

[16] Bingul Z, Gul K. Intelligent-PID with PD feedforward trajectory tracking control of an autonomous underwater vehicle. Machines. 2023;11:300.

[17] Chen L, Xu R. Deep reinforcement learning for autonomous underwater vehicle control. J Marine Sci Eng. 2023;12(7):777.

[18] Zhang Y, Liang J. Advanced navigation systems for autonomous underwater vehicles: Challenges and opportunities. J Marine Sci Eng. 2023;12(1):112.

[19] Tian Y, Xu J. Lyapunov stability and dynamic position control for AUVs: A comprehensive review. Marine Technol Soc J. 2023;57(1):15-28.

[20] Santos CM, Costa P. Lyapunov function-based control for underwater robot dynamic positioning. Ocean Eng. 2023;277:113197.

[21] Kumar S, Shah R. Decentralized control strategies for underwater robot swarms. J Marine Sci Eng. 2023;12(1):568.

[22] Liu L, Zhang L, Pan G, Zhang S. Robust yaw control of autonomous underwater vehicle based on fractional-order PID controller. Ocean Eng. 2022;257:111493.

[23] Shen C, Shi Y, Buckham B. Lyapunov-based model predictive control for dynamic positioning of autonomous underwater vehicles. In: Proceedings of ICUS. 2017:8278413.

[24] Hasan MW, Abbas NH. Disturbance rejection for underwater robotic vehicle based on adaptive fuzzy with nonlinear PID controller. ISA Trans. 2022;130:360-376.

[25] Lv T, Zhou J, Wang Y, Gong W, Zhang M. Sliding mode-based fault tolerant control for autonomous underwater vehicle. Ocean Eng. 2020;216:107855.

[26] Khan S, Ahmed S. Lyapunov-based dynamic position control for underwater vehicles in complex environments. Int J Control Autom Syst. 2023;21(4):1023-1036.

[27] Li J, Zhou T. Underwater power conversion and junction technology for underwater wireless power transfer stations. J Marine Sci Eng. 2023;12(4):561.

[28] Sun H, Wang Q. Model predictive control of an underwater robot with fuzzy logic for object tracking. J Marine Sci Eng. 2023;12(3):356.

[29] Yan Z, Wang M, Xu J. Robust adaptive sliding mode control of underactuated autonomous underwater vehicles with uncertain dynamics. Ocean Eng. 2019;173:802-809.

[30] Antonelli G, Fossen TI, Yoerger DR. Modelling and control of underwater robots. In: Siciliano B, Khatib O, eds. Springer Handbook of Robotics. Cham, Switzerland: Springer; 2016.

[31] Fern´andez F, Mart´ınez R. Robust control strategies for dynamic positioning of AUVs using Lyapunov theory. J Control Autom Electr Syst. 2023;34(1):54-67.

[32] Wang Y, Wu Z, Cheng Y. Dynamic modelling and control of an underwater vehicle. Ocean Eng. 2021;220:108188.

[33] Song F, An PE, Folleco A. Modelling and simulation of autonomous underwater vehicles: Design and implementation. IEEE J Ocean Eng. 2003;28(2):283-296.

[34] Bian XQ, Fu MY, Wang YH. The Dynamic Positioning of the Ship. Beijing, China: Science Press; 2011:5-9.

[35] Fossen TI, Grovlen A. Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping. IEEE Trans Control Syst Technol. 1998;6:121-128.

[36] Wang N, Liu ZZ, Zheng ZJ, Er MJ. Global exponential trajectory tracking control of underactuated surface vehicles using dynamic surface control approach. In: Proceedings of the 2018 International Conference on Intelligent Autonomous Systems; Singapore. 2018:1-3.

[37] Vu MT, Le TH, Thanh HLNN, et al. Robust position control of an over-actuated underwater vehicle under model uncertainties and ocean current effects using dynamic sliding mode surface and optimal allocation control. Sensors. 2021;21:747.

[38] Vu MT, Thanh HLNN, Huynh TT, et al. Station keeping control of a hovering over-actuated autonomous underwater vehicle under ocean current effects and model uncer- tainties in horizontal plane. IEEE Access. 2021;9:6855-6867.

[39] Hu CD, Wu DF, Liao YX, Hu X. Sliding mode control unified with the uncertainty and disturbance estimator for dynamically positioned vessels subjected to uncertainties and unknown disturbances. Appl Ocean Res. 2021;109:102564.

[40] Rojsiraphisal T, Mobayen S, Asad JH, et al. Fast terminal sliding control of under- actuated robotic systems based on disturbance observer with experimental validation. Mathematics. 2021;9:1935.

[41] Thanh HLNN, Vu MT, Mung NX, Nguyen NP, Phuong NT. Perturbation observer- based robust control using multiple sliding surfaces for nonlinear systems with influences of matched and unmatched uncertainties. Mathematics. 2020;8:1371.

[42] Mcgann C, Py F, Rajan K, Ryan J, Henthorn R. Adaptive sampling for deep-sea ecology investigations. In: Proceedings of the IEEE/MTS OCEANS Conference; Seattle, WA. 2019:1-5.

[43] Liu P, Xiao W. Design and testing of an integrated power and propulsion system for a hybrid underwater glider. J Ocean Eng Sci. 2023;8:147-156.

[44] T. Xu, J. Liu, Z. Zhang, G. Chen, D. Cui and H. Li, ”Distributed MPC for Trajectory Tracking and Formation Control of Multi-UAVs With Leader-Follower Structure,” in IEEE Access, vol. 11, pp. 128762-128773, 2023

[45] T. Gao et al., ”Model Predictive Control for an Autonomous Underwater Robot with Fully Vectored Propulsion,” 2024 IEEE International Conference on Robotics and Au- tomation (ICRA), Yokohama, Japan, 2024

[46] T. Baca, D. Hert, G. Loianno, M. Saska and V. Kumar, ”Model Predictive Trajectory Tracking and Collision Avoidance for Reliable Outdoor Deployment of Unmanned Aerial Vehicles,” 2018

[47] B. M. Patel and S. K. Dwivedy, “3D dynamics and control of a snake robot in uncertain underwater environment,” *Robotica*, pp. 1–28, 2024. doi:10.1017/S0263574724000821

Appendix

```
function axp(Position, srf_in)

    hold off
    srf_in = srf_in + repmat([Position(1) Position(2) Position(3)],[16 1]);
    h=plot3(srf_in(:,1),srf_in(:,2),srf_in(:,3),'b');
    ax = gca;
    ax.XLim = [-15 15];
    ax.YLim = [-15 15];
    ax.ZLim = [-15 15];
    xlabel('X (m)');
    ylabel('Y (m)');
    zlabel('Z (m)');
    grid on
end
```

```
function [u1, u2, u3] = mpc(r1, d1, r2, d2, r3, d3, t)

    u1 = zeros(size(t));
    u2 = zeros(size(t));
    u3 = zeros(size(t));
    delta = 0.1;
    N = 5;
    Q = diag([10^6, 10^6, 10^5, 10^3, 10^3, 10^3]);
    R = diag([10^-4, 10^-4, 10^-4, 10^-4]);
    P = diag([10^4, 10^4, 10^3, 10, 10, 10]);
    Kp1 = 1; Ki1 = 0.001; Kd1 = 1.1;
    Kp2 = 0.75; Ki2 = 0.0005; Kd2 = 1.1;
    Kp3 = -50; Ki3 = -0.001; Kd3 = -30;
    integral1 = 0; prev_error1 = 0;
    integral2 = 0; prev_error2 = 0;
    integral3 = 0; prev_error3 = 0;
    u1 = zeros(size(t));
    u2 = zeros(size(t));
    u3 = zeros(size(t));
    for k = 1:length(t)
        ref1 = r1(k); dist1 = d1(k);
        ref2 = r2(k); dist2 = d2(k);
        ref3 = r3(k); dist3 = d3(k);
        error1 = ref1 - dist1;
        error2 = ref2 - dist2;
        error3 = ref3 - dist3;
        integral1 = integral1 + error1 * delta;
        derivative1 = (error1 - prev_error1) / delta;
        u1(k) = 0*(Kp1 * error1 + Ki1 * integral1 + Kd1 * derivative1);
        integral2 = integral2 + error2 * delta;
```

```
        derivative2 = (error2 - prev_error2) / delta;
        u2(k) = 0*(Kp2 * error2 + Ki2 * integral2 + Kd2 * derivative2);
        integral3 = integral3 + error3 * delta;
        derivative3 = (error3 - prev_error3) / delta;
        u3(k) = 0*(Kp3 * error3 + Ki3 * integral3 + Kd3 * derivative3);
        prev_error1 = error1;
        prev_error2 = error2;
        prev_error3 = error3;
    end
    Kp1 = 1; Ki1 = 0.001; Kd1 = 1.1;
    Kp2 = 0.75; Ki2 = 0.0005; Kd2 = 1.1;
    Kp3 = -50; Ki3 = -0.001; Kd3 = -30;
    M = diag([1, 1, 1, 1, 1, 1]);
    Kp = diag([Kp1, Kp2, Kp3]);
    V = @(x, u) 0.5 * M * x + 0.5 * (u' * Kp * u);
    ref1 = r1(k); dist1 = d1(k);
    ref2 = r2(k); dist2 = d2(k);
    ref3 = r3(k); dist3 = d3(k);
    error1 = ref1 - dist1;
    error2 = ref2 - dist2;
    error3 = ref3 - dist3;
    final_state = [r1(end); r2(end); r3(end); u1(end); u2(end); u3(end)];
    final_control = [u1(end); u2(end); u3(end)];
    lyapunov_value = V(final_state, final_control);
end
```

```
function Visualize(Position, srf_in)
    coder.extrinsic('axp');
    axp(Position, srf_in);
end
```